Methods and Algorithms of Alternatives Ranging in Managing the Telecommunication Services Quality

Pham Quang Hiep
Faculty of Informational Technologies and Communications
Astrakhan state technical university, Astrakhan, Russia

Kvyatkovskaya Irina Yurievna
Faculty of Informational Technologies and Communications
Astrakhan state technical university, Astrakhan, Russia

Shurshev Valery Fedorovich
Faculty of Informational Technologies and Communications
Astrakhan state technical university, Astrakhan, Russia

Popov Georgy Alexandrovich
Faculty of Informational Technologies and Communications
Astrakhan state technical university, Astrakhan, Russia

Abstract
The article deals with methods of solving the problem of ranging of alternatives in information-analytical system of managing the quality of telecommunication services rendering process. Tasks of choice are determined, in which the alternatives are as follows: states of quality of different objects in the structure of telecommunication company management. An algorithm of ranging of objects is elaborated for the case of using unstructured set of indices. The algorithm enables to determine the objects priorities and to select the best ones among them. The suggested methods may be used while elaborating the programs of improvement of telecommunication companies competitiveness.

Keywords: quality index, telecommunication service, set of indices, ranging, factor-set, associative matrix, alternative, Pareto layer, meta-index, Kemeni median.

1. Introduction
Due to information technologies development the market of telecommunication services (TCS) is permanently developing; during the last decade telecommunication services became one of the key sectors of developed countries economy and began to play more and more large role in socio-economic sphere of society.

At the same time, during the last time there appeared many companies, which take part in rendering the TCS. However, each company aiming at market success should improve effectiveness of management process in order to satisfy clients’ needs. Ranging of objects [1-17] while assessing the quality of rendering the services is one of the most important component of effective management in this field.

Thus, the aim of this article is to elaborate methods of ranging of objects; the methods participating in rendering TCS on the base of formalization, as well as the choice of mathematical tools.

While assessing the quality of rendering of TCS, let’s define a set of quality assessments achieved while TCS quality monitoring, as a set of points of criteria space, the points having
in formal view a criterial presentation. Space of indices (indicators) of the process of TCS quality assessment may be shown in the figure 1.

![Figure 1. The system of coordinates of multi-dimension information space](image)

Projection of elements of multi-dimension information space enables to form selection (sampling) of data for solving of the following tasks of alternatives ranging:

- Task of ranging of quality states for different objects in the given moment of time $t^*(Ob_i, k_j), i = 1, n, j = 1, m$;
- Task of ranging of quality states for one object on different time moments $Ob^*(t_g, k_j), g = 1, s, j = 1, m$;
- Task of ranging of objects according to the given index $k^*(Ob_i, t_g), i = 1, n, g = 1, s$.

There are many methods to solve the problems of ranging objects. However, most of them based entirely on the awareness of the decision-making person (DMP). In other words, these methods cannot completely solve the problem, when the DMP do not have sufficient information on various decision levels. Therefore, in this article, we propose new methods to solve the problem of ranging objects with different levels of awareness of decision makers. We use the classification was introduced by Kandyrin Y.V. [8, 9], in which types of awareness DMP were separated: fully, poorly and average informed.

**Fully informed** DMP is able to single out quality indicators and to set priorities among them. Most methods of decision-making are designed for fully informed decision makers and widely known [5, 6, 13, 14, 17].

**Average informed** DMP are not able to unequivocally establish the linear order on a set of quality parameters of compared objects, but can identify a set of quality objects parameters and their priorities for the partial order in the adopted system of meta-indices of quality. In this case, the method is used to rank the formation of meta-indices on the set of initials index-decision criteria.

**Poorly informed** DMP can only allocate a set of quality indicators, but cannot set up priorities among them. In that case, generated a set of non-dominated alternatives incomparable with Pareto set, and consists frequently from a significant number of alternatives, which hinders for decision-making. The authors propose the use of distribution median method for narrow the set of optimal alternatives.
2. **Solving the task using the set of meta-indices.**

In priority, let’s define an object (subject, which participates in rendering the TCS) as an alternative in the task of multi-criterial decision making [1, 2, 3, 12, 15, 16]. All the given indices which characterize various sides of this process, must be brought in to a system. One part of indices reflects performance parameters of technical system, another part is formed by processing of data which are stored in enterprise’s corporative information system, the others are to be formed by expert way or in result of inquiry.

Let us assume, there are \( n \) objects, each of them is characterized by \( m \) indices. We designate value of the \( j \)-th index for choice of the \( i \)-th object as: \( k_{ij}, j=1, m, i=1, n \).

The algorithm includes the following steps:

1. Define a set of initial data – totality of informative indices of quality \( \{k_1, k_2, ..., k_m\} \), on which the comparison of objects from the set \( OBJ = <Ob_1, Ob_2, ..., Ob_n> \) will be done. It is proposed that these indices weakly correlate between each other – i.e. they carry different information.

2. We choose generalized meta-indices \( (K_1, K_2) \) and find coordinates of each index on the plane \( K_1OK_2 \).

3. We build factor-set on the base of information about linear orders \( L(OBJ/k_j), j=1, m \) on the indices of quality \( \{k_j\} \).

Let us give definition of environs \( O_i(OBJ/k_j) \) in the factor-set for quality index \( k_j \) for the ratio \( \succ = \):

\[
O_i(OBJ/k_j) = \{Ob_p : k_j(Ob_r) \leq k_j(Ob_p), Ob_p, Ob_r \in OBJ, Ob_r \succ = Ob_p \}, p=1, n.
\]

Then the factor-set \( OBJ/k_j \) may be presented as a totality of environs built for all elements of the set \( OBJ: OBJ/k_j = \{O_i(OBJ/k_j)\}, i \in \{1, ..., |OBJ|\} \).

4. Let us build \( n \) linear orders \( L(OBJ/k_i), i=1, n \)

5. Let’s build associative matrices of factor-sets \( AM_i, i=1, n \), for each linear order (Table 1). In them, each column determines the environs \( O_i(Ob_i) \) of the \( i \)-th object and includes a set of all dominating and equivalent objects for it. The element \( am_{i,p} \) of the associative matrix is defined as follows:

\[
am_{i,p} = \begin{cases} 0, & Ob_p \succ Ob_i, Ob_{i,p} \in L(OBJ/k_j), i \neq p, \\ 0, & i = p, \\ 1, & Ob_p \prec Ob_i, Ob_{i,p} \in L(OBJ/k_j), i \neq p. \end{cases}
\]

<table>
<thead>
<tr>
<th>Alternatives/environments</th>
<th>( O_1(Ob_i/k_j) )</th>
<th>( O_2(Ob_i/k_j) )</th>
<th>( .... )</th>
<th>( O_n(Ob_i/k_j) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Ob_1 )</td>
<td>0</td>
<td>( am_{12} )</td>
<td>( .... )</td>
<td>( am_{1n} )</td>
</tr>
<tr>
<td>( Ob_2 )</td>
<td>( am_{21} )</td>
<td>0</td>
<td>( .... )</td>
<td>( am_{2n} )</td>
</tr>
<tr>
<td>( ... )</td>
<td>( .... )</td>
<td>( .... )</td>
<td>( .... )</td>
<td>( .... )</td>
</tr>
<tr>
<td>( Ob_n )</td>
<td>( am_{n1} )</td>
<td>( am_{n2} )</td>
<td>( .... )</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1. Associative matrix \( AM \) for the factor-set \( \Phi_{OBJ/k_i} \) of linear order

6. Let’s define the co-ordinates of indices in the space of over-system indices \( (K_1, K_2) \) by expert way.

7. Let’s distribute indices on Pareto layer. For all indices of one Pareto layer, let’s find a cross of associative matrices according to \( \pi \)-rule[8, 9]: Let’s assume: there is given a set of numbers of indices, included in the \( p \)-th Pareto layer:

\[
\Phi_{OBJ}(k_1, k_2, ..., k_p) = \Phi_{OBJ/k_1} \cap \Phi_{OBJ/k_2} \cap ... \cap \Phi_{OBJ/k_p}.
\]
8. If there is an indistinguishability of objects in associative matrices $AM_i$, then we can define an indistinguishability of objects for any agreed $L$-criterion [8, 9].

For this aim the environs of indistinguishable (indiscernible) objects are compared inside initial associative matrices $AM_i$: if $am_{i,p}^{r,s} = am_{p,r}^{i,s} = 1$, then $am_{i,p}^{r,s}$ is an indistinguishable element. According to the $L$-rule all the elements of the $AM$ matrix which conform to factor-set $\Phi_{OBJ}/\{k_1, k_2, \ldots, k_p\}$ are to be transferred into resulting matrix, except indistinguishable elements. On the places of these indistinguishable elements the resulting matrix will have a result of conjunction of: this element $am_{i,p}^{r,s}$ and the matrix element corresponding to the factor-set $\Phi_{OBJ}/\{k_1, k_2, \ldots, k_p\}$. In result of such operation we can get a resulting (final) associative matrix of more high order for totality of quality indices (table 2).

<table>
<thead>
<tr>
<th>Environments / Alternatives</th>
<th>$O_1(Obi)$</th>
<th>$O_2(Obi)$</th>
<th>$\ldots$</th>
<th>$O_n(Obi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ob_1$</td>
<td>0</td>
<td>$G_{12}$</td>
<td>$\ldots$</td>
<td>$G_{1n}$</td>
</tr>
<tr>
<td>$Ob_2$</td>
<td>$G_{21}$</td>
<td>0</td>
<td>$\ldots$</td>
<td>$G_{2n}$</td>
</tr>
<tr>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
</tr>
<tr>
<td>$Ob_n$</td>
<td>$G_{n1}$</td>
<td>$G_{n2}$</td>
<td>$\ldots$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2. Resulting associative matrix

The Alternative $Ob_i$ is included into the Pareto set of optimal decisions in case if the condition is true for the environ $O_{p,1}(Ob_i/\{k_j\})$ of the alternative $Ob_i$:

$$\bigvee_{i=1}^{n} G_{i,p} = 0 \quad (3)$$

9. It is possible to range the objects of the set $OBJ$ according to the resulting matrix, it enables to determine priorities of objects and choose the best among them. Incomparable alternatives, for which $G_{i,p}G_{p,i} = 1$, form the Pareto layers.

In result of algorithm’s work the initial set of alternatives comes to lexicographical putting in good order.

**Example:** use of this method for solving a task of quality states ranging for different objects in given moment of time $t^*(Ob_i, k_j), i = 1, n, j = 1, m$.

Let there be an initial set of five objects – five companies-providers of TCS, for each of them 3 indices are assumed: “Time of fulfilling of initial consumer’s connection to the network (Day)”, “Percentage of successful calls of a consumer (Percent)”, «Degree of consumers’ satisfaction with quality of technical maintenance of a service (Score, points)» (Table 3)

It is necessary to put in order the objects by decrease of the indication. We will consider an object with maximal (or minimal) values of indices to be the best one (signature: direction of an arrow ↑ and ↓): $(k_1 \downarrow, k_2 \uparrow, k_3 \uparrow)$.

<table>
<thead>
<tr>
<th>Objects</th>
<th>$k_1$ (date) ↓</th>
<th>$k_2$ (percent) ↑</th>
<th>$k_3$ (score) ↑</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ob_1$</td>
<td>6</td>
<td>90</td>
<td>3</td>
</tr>
<tr>
<td>$Ob_2$</td>
<td>4</td>
<td>98</td>
<td>3,5</td>
</tr>
<tr>
<td>$Ob_3$</td>
<td>5</td>
<td>96</td>
<td>4,5</td>
</tr>
<tr>
<td>$Ob_4$</td>
<td>3</td>
<td>98</td>
<td>4</td>
</tr>
<tr>
<td>$Ob_5$</td>
<td>2</td>
<td>97</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3. Initial data
2. We choose generalized meta-indices of quality: $K_1 = 1/\text{Importance}$, $K_2 = \{\text{Cost}\}$.

3. Let’s assign priorities to indices in the given space of meta-indices:

$$K_1 = \frac{1}{\text{Importance}}, \quad K_2 = \{\text{Cost}\}.$$ 

4. We build linear orders according to $k_1, k_2, k_3$:

$$L(OBJ/k_1) = \langle Ob_5, Ob_4, Ob_2, Ob_3, Ob_1 \rangle;$$

$$L(OBJ/k_2) = \langle \{ Ob_2, Ob_4 \}, Ob_5, Ob_3, Ob_1 \rangle;$$

$$L(OBJ/k_3) = \langle Ob_5, Ob_3, Ob_4, Ob_2, Ob_1 \rangle.$$ 

5. Let’s build three associative matrices for each linear order (Table 4a, 4b, 4c).

### Table 4a. Associative matrix $AM_1$ for factor-set $\Phi_{OBJ/k_1}$

<table>
<thead>
<tr>
<th>Obj/env</th>
<th>$O_1(Ob_1/k_1)$</th>
<th>$O_2(Ob_2/k_1)$</th>
<th>$O_3(Ob_3/k_1)$</th>
<th>$O_4(Ob_4/k_1)$</th>
<th>$O_5(Ob_5/k_1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ob_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Ob_2$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Ob_3$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Ob_4$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Ob_5$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 4b. Associative matrix $AM_2$ for factor-set $\Phi_{OBJ/k_2}$

<table>
<thead>
<tr>
<th>Obj/env</th>
<th>$O_1(Ob_1/k_2)$</th>
<th>$O_2(Ob_2/k_2)$</th>
<th>$O_3(Ob_3/k_2)$</th>
<th>$O_4(Ob_4/k_2)$</th>
<th>$O_5(Ob_5/k_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ob_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Ob_2$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$Ob_3$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Ob_4$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$Ob_5$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 4c. Associative matrix $AM_3$ for factor-set $\Phi_{OBJ/k_3}$

<table>
<thead>
<tr>
<th>Obj/env</th>
<th>$O_1(Ob_1/k_3)$</th>
<th>$O_2(Ob_2/k_3)$</th>
<th>$O_3(Ob_3/k_3)$</th>
<th>$O_4(Ob_4/k_3)$</th>
<th>$O_5(Ob_5/k_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ob_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Ob_2$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Ob_3$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$Ob_4$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Ob_5$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
6. Decision begins with the index \( k_2 \), associative matrix for it is built; further - \( k_1 \) and \( k_3 \)
take part, they are incomparable between themselves.

7. Let’s build an associative matrix \( AM_{13} \) for factor-set \( \Phi_{OBJ}/\{k_1, k_3\} \), elements of the
factor-set are got by means of making Boolean conjunction operation on matrix elements \( AM_1 \) и \( AM_3 \):

\[
\Phi_{OBJ}/\{k_1, k_3\} = \Phi_{OBJ}/k_1 \cap \Phi_{OBJ}/k_3.
\]

<table>
<thead>
<tr>
<th>Objects/ environs</th>
<th>( O_1(Ob_1) )</th>
<th>( O_2(Ob_2) )</th>
<th>( O_3(Ob_3) )</th>
<th>( O_4(Ob_4) )</th>
<th>( O_5(Ob_5) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Ob_1 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Ob_2 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Ob_3 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Ob_4 )</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Ob_5 )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5. Associative matrix \( AM_{13} \) for factor-set \( \Phi_{OBJ}/\{k_i, k_j\} \)

8. There are incomparable elements in the matrix \( AM_2 : \ am_{2,4}^2 = am_{4,2}^2 = 1 \), so we form a
resulting set, which settles the order on \( OBJ \) by means of crossing the matrix \( AM_{12} \) and
matrix \( AM_1 \) according to the \( L \)-rule by choosing undistinguishable elements from the matrix
\( AM_3 \). It is also \( am_{2,4}^2 = am_{4,2}^2 = 1 \). Lets find the conjunction of elements \( (2,4) \) \( (2^{nd} \ line, 4^{th} \ column) \) and \( (4,2) \), which occupy similar places in the matrix \( AM_{13} \) and \( AM_2 \) and lets put them into new matrix \( AM_{123} \):

\[
am_{2,4}^{13} = am_{2,4}^2 \cap am_{4,2}^2 = 0 \cap 1 = 0; \am_{4,2}^{13} = am_{4,2}^2 \cap am_{4,2}^2 = 1 \cap 1 = 1.
\]

Other elements are distinguishable and go from the matrix \( AM_2 \) into resulting matrix
\( AM_{123} \) (Table 6).

<table>
<thead>
<tr>
<th>Objects/ environs</th>
<th>( O_1(Ob_1) )</th>
<th>( O_2(Ob_2) )</th>
<th>( O_3(Ob_3) )</th>
<th>( O_4(Ob_4) )</th>
<th>( O_5(Ob_5) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Ob_1 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Ob_2 )</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( Ob_3 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Ob_4 )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( Ob_5 )</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6. Resulting associative matrix \( AM_{123} \)

9. Let’s detect relation of the order on the resulting matrix:

\[
L(OBJ/\{k_1, k_3, k_3\}) = \langle Ob_4, Ob_2, Ob_5, Ob_3, Ob_1 \rangle.
\]

3. Solution of the task using median distribution

This algorithm includes the following steps:

1. Choose a complex of informative indices of quality \( \{k_1, k_2, ....., k_m\} \), according to which
we’ll assess each object from the set \( OBJ = \langle Ob_1, Ob_2, ....., Ob_n \rangle \)

2. Range the objects onto each line corresponding to one of the indices. In each ranging
the first place is the most attractive. Let’s re-define all the assessments of objects in the order scale.
Each \( j \)-th index gives its vector of preferences \( \lambda_j = (\lambda_{j1}, \lambda_{j2}, ....., \lambda_{jm}) \), \( j = 1, m \), where
\( \lambda_{j\gamma} \) is the ordinal number of the object, which has the i-th place in the ranging on \( j \)-th index.
Then let’s unite received \( n \) rangings \( K = \{ \lambda_1, \lambda_2, \ldots, \lambda_m \} \), into the matrix \( K \), columns of which correspond to assessments of each object \( \pi_j = (\pi_{j1}, \pi_{j2}, \ldots, \pi_{jn}) \) and are expressed by ranges.

3. Build a matrix of losses \( R = \{r_{pq}\} \), where \( r_{pq} = d(\pi_p, \pi'_q) \), \( j = 1, m \). Consider vectors, in which the direction with number \( i \) (\( i = 1, n \)) is located consecutively from the 1st till the \( n \)th place: \( \pi = (\pi_1, \pi_2, \ldots, \pi_p, \ldots, \pi_n) \) - ranging in which the \( p \)-th object stands in the \( q \)-th place (i.e. \( \pi_p = q - 1 \)), \( d(\pi_p, \pi'_q) \) - distance between two rangings, it is defined by the formula of Kemeni-Snell median \([1, 4, 6, 10, 11]\):

\[
d(\pi_p, \pi'_q) = \sum_{j=1}^{m} |\pi_i - \pi'_i| \quad (4)
\]

4. By minimization of functional we solve a task of destinations:

\[
\begin{align*}
\sum_{p=1}^{n} \sum_{q=1}^{n} r_{pq} \times x_{pq} & \rightarrow \min, \quad x_{pq} \geq 0 \\
\sum_{p=1}^{n} \sum_{q=1}^{n} x_{pq} = 1, \quad p = 1, n, \sum_{q=1}^{n} x_{pq} = 1, \quad q = 1, n
\end{align*}
\]

(5)

where \( X = \{x_{pq}\} \) is a binary matrix of destinations: \( x_{pq} = 1 \) if \( p \)-th object occupies the \( q \)-th place; otherwise \( x_{pq} = 0 \).

If condition (5) is observed the matrix \( X \) corresponds to some ranging.

For matrix \( X \), let’s restore a vector of group preference \( K^* \), analyzing the matrix \( X \) on lines: if \( x_{pq} = 1 \), then in the vector \( K^* \) we assume \( k^*_q = p \). And we receive the ranging \( \{ k^*_1, k^*_2, \ldots, k^*_n \} \), in which \( k^*_j \) indicates the range of the \( j \)-th object.

The task of destination may be solved using the methods of linear programming or the algorithm of solving of transport task. In this case to solve the task of destination with minimal cost we can use the application «Excel», option «Search and solving».

5. If it is necessary to define coefficients of preferences of objects, it is possible to use the method of pare comparison or the scheme of Fishburn scales.

For the method of pair comparison: lets form a matrix of pair comparison \( L = \{ w_{pq} \} \), \( p, q = 1, n \) for a group preference. Its elements are defined as follows:

\[
\begin{align*}
w_{pq} &= 2 \quad \text{if element } p \text{ is more preferable, than element } q; \\
w_{pq} &= 1 \quad \text{if elements } p \text{ and } q \text{ are preferable equally;} \\
w_{pq} &= 0 \quad \text{if element } p \text{ is less preferable, than element } q.
\end{align*}
\]

Then we calculate the sum of elements of each line \( u_p = \sum_{q=1}^{n} w_{pq} \) and value \( V = \sum_{p=1}^{n} u_p \).

Then we find values corresponding to each object:

\[
X_p = \frac{u_p}{V}, \quad p = 1, n
\]

(6)

**Example:** use of this method for solving the task of ranging of quality states for one object in different moments of time \( \text{Obj}^*(t_s, k_j), \quad g = 1, s, \quad j = 1, m \); to assess states of quality of services rendered by the Viettel company in quarters of 2014 according to 5 indices:

- \( k_1 \) – time of fulfilling of consumer’s initial connection to a network (day);
- \( k_2 \) – percentage of successful calls of a consumer (percent);
- \( k_3 \) – speed of fulfilling of connection (second);
- \( k_4 \) – quality of transmission of voice (score);
- \( k_5 \) – degree of satisfaction (score).

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( k_1 \downarrow )</td>
</tr>
<tr>
<td>( t_1 )</td>
<td>3</td>
</tr>
<tr>
<td>( t_2 )</td>
<td>6</td>
</tr>
<tr>
<td>( t_3 )</td>
<td>3</td>
</tr>
<tr>
<td>( t_4 )</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 7. Initial data

Priority of each index is defined as follows:
\( \lambda_1 = (0, 2, 0, 1) \), \( \lambda_2 = (2, 3, 0, 1) \), \( \lambda_3 = (1, 1, 0, 2) \), \( \lambda_4 = (0, 1, 0, 2) \), \( \lambda_5 = (1, 2, 0, 2) \).

\( \pi_1 = (0, 2, 1, 0, 1) \); \( \pi_2 = (2, 3, 1, 2) \); \( \pi_3 = (0, 0, 1, 0) \); \( \pi_4 = (1, 1, 2, 0) \).

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 8. Matrix of losses \( R \)

Analyzing the matrix of destination: \( x_{12} = 1 \); \( x_{24} = 1 \); \( x_{31} = 1 \); \( x_{43} = 1 \), we receive \( K^* = (3, 1, 4, 2) \).

<table>
<thead>
<tr>
<th>q1</th>
<th>q2</th>
<th>q3</th>
<th>q4</th>
<th>( u_p = \sum_{q=1}^{n} w_{pq} )</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>5/16=0,312</td>
</tr>
<tr>
<td>p2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1/16=0,062</td>
</tr>
<tr>
<td>p3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>7/16=0,438</td>
</tr>
<tr>
<td>p4</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>3/16=0,188</td>
</tr>
</tbody>
</table>

Table 10. Matrix of pair comparison \( L \)

Resulting rangings of alternatives (of 2014 quarters) according to the method of searching of Kemeni median are presented on the order:
\( t_3(0,438) > t_1(0,312) > t_4(0,188) > t_2(0,062) \)

4. Conclusion

The article describes methods of solving the task of both assessment and ranging of objects in telecommunication companies according to multi-criterial choice, that enables to compare and evaluate quality of rendering the services between companies in whole and telecommunication companies particularly. Besides, such approach may be applied for example for comparison of different subjects of economy, which provide telecommunication services: mobile phone companies and their branches (offices). The proposed algorithms
allow the ranging of objects under conditions of incomplete awareness decision maker, when there is no information about the priorities of selection criteria for alternatives.

References


