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REASONING ABOUT THE GAME „CLUE“ BY USING OTTER

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Abstract: *In this article the possibilities of reasoning about the card version of the game Clue by using OTTER - system for automatic theorem proving have been presented. The game Clue, as game based on knowledge have been modelled by PVETO logic - propositional multi-modal epistemic logic with temporal parameter adapted for reasoning with OTTER. PVETO logic is an extension of S5m logic and it's most important characteristics are the introduction of special derivation predicates for every participant in the card game and introduction of temporal parameter. Temporal parameter refers to the moment of time in which we follow the truthfulness of the epistemic formulae.*

Keywords: *card games, Clue, epistemic logic, PVETO logic, automated reasoning, OTTER.*

1. THE GAME "CLUE"

It In the game Clue the players gather information about a murder. A man has been found dead at his mansion. The players must determine who has done it, where and with what weapon. The suspects, the possible vehicle and destination of crime are displayed on the cards. There are nine possible rooms in the mansion, six suspects and six possible murder weapons. The deck of cards contains one card for each of the suspects, weapons and rooms. The commercial version of the game involves a game board which represents nine rooms in the mansion and access between them, six character pawns, each representing one of the suspects and two dices to move pawns through the game board. The original and complete equipment and rules of the game Clue is shown in [1].

In this article we ignore some aspects and some rules of the game as we are interested in representing and reasoning about knowledge. Also, we reduce the game to illustrate how the actions in game can be specified. Let us assume that there are simply four suspects (Professor_Plum, Mr_Green, Mrs_Peacock, Miss_Scarlett), four weapons (Knife, Candlestick, Pistol, Rope) and no rooms. So, we have eight cards. It is easy to scale this up to the full number of suspects, weapons and rooms. One card representing a suspect and one a weapon is separated from the deck of cards and put on the table upside down so that none of the players see which cards they are. This pair of cards represents a real murderer and a weapon. In the original version of the game the remaining cards are mixed together and are equally dealt out amongst the players. For the purpose of further simplification we will develop a version where the cards are not mixed together but instead each player receives one card representing a suspect and one a weapon. The players are making suggestions that the crime was committed by that suspect and with that weapon. If the player left to the player who has voiced his suggestion has one of the suggested cards, then he shows a card to the player who has given the suggestion in a way that the other players don't know which card it is. He privately shows always only one card irrespective of whether he has one or both cards. If the player who is left to the player who gives the suggestion does not have in his hand any of the suggested cards, then he announces this and the following player does the same: privately shows one of the suggested cards or announces that he does not have any. This continues until some player does show a card privately or the round is not finished, and any players do not show any cards. In our version of the game it is clear that the round has finished when at most two players answer the default suggestion. In the following round the player who is right of the player who gave the suggestion gives a new suggestion and so on. The players apply their knowledge about the cards and all other knowledge which they get about the knowledge of the other players in order to discover the murderer and the weapon. When a player solves the mystery he proclaims it and that player is the winner. The player who brings out the suggestion is not allowed to suggest pair of cards which they hold in their hand. Also, the player may not suggest the same cards twice in one game, because by doing so they disable other players to come to any new information. A player can repeat suggestion only in the case if he deduces which cards are on the table.

2. MODELLING THE GAME "CLUE"

2.1. ASSUMPTIONS AND BASIC PROPOSITIONS

In the game Clue there are three active players a1, a2 and a3, as well as card table as a fourth player a0. Player a0 is a player without knowledge and is not able to take any action, but has the cards which represent the actual murderer and weapon. In this way we can, without loss of generality, assume that all cards are dealt between the players. Also, without loosing generality we can assume the following basic propositions:

1. Professor Plum made a crime with the Knife (player a0 has that cards)
2. Player a1 has the cards Mr_Green and Candlestick
3. Player a2 has the cards Mrs_Peacock and Pistol
4. Player a3 has the cards Miss_Scarlett and Rope

Also, we can suppose that the order the players give their suggestions is a1, a2, a3, then a1 again and so on. It follows that the players answer the suggestions in the opposite order. For example if a player a1 gives a suggestion then player a3 answer first and then eventually player a2.

2.2. LANGUAGE OF PVETO LOGIC ADAPTED FOR THE GAME "CLUE"

As the foundation for formalising and reasoning about the knowledge of players in the game Clue the multi-modal epistemic logic S5m can be used [8]. However, S5m logic is not enough for our needs and we employ PVETO logic - propositional multi-modal epistemic logic with temporal parameter adapted for reasoning with OTTER. PVETO logic is introduced in [6, pp. 45-52] and it's most important characteristics are the introduction of special derivation predicates for every participant in the card game and introduction of temporal parameter as we describe below.

The language of PVETO logic adapted for our needs in this article is comprised of the following sets of symbols:

1. Finite set of constants A={a0,a1,...,an} which represent the players
2. Finite set of constants C which represent the cards
3. A set of variables V (strings which start from u, v, w, x, y or z)
4. Binary function symbol "d" where we interpret the expression d(x,y) as "player x has the card y"
5. Unary function symbol "n" where we interpret the expression n(x) as $\neg x$ in classical propositional logic
6. Binary function symbol "i" where we interpret the expression i(x,y) as $x \rightarrow y$ in classical propositional logic
7. Finite set of knowledge operators K={K1,K2,...,Kn} for players where we interpret the expression Ki(x,v) as "player ai knows formula x in moment of time v"
8. Finite set of binary predicate symbols P={P1,P2,...,Pn} where we interpret the expression Pi(x,w) as "formula x is derivable for player ai in moment of time w"[1]

Temporal parameter which is introduced at points 7 and 8 can have the values 0,1,2,... in accordance with the fact that we consider time in the case of the game Clue to be a discrete sequence of moments. This is because the knowledge of the players is changed only as a result of their actions. In fact, through time the knowledge of players is changed so that it is necessary to distinguish the truthfulness of epistemic formulae in different moments of time. This is why in the PVETO logic the temporal parameter is introduced. Temporal parameter refers to the moment of time in which we follow the truthfulness of the epistemic formulae. In PVETO logic it is assumed that players never forget, in that they don’t lose the knowledge which they have formed in whatever past moment of time. This characteristic is introduced as axiom and is formalized below.

From the symbols 1-8 we build expressions which we call atomic formulae. From the atomic formulae, the symbol "-" (as an operator of negation) and symbol "|" (as an operator of disjunction) we build clauses. Operators "-" and "|" are applicable only to the atomic formulae. [5, pp. 95-101]

2.3. AXIOMS AND DERIVATION RULES OF PVETO LOGIC

The axioms and derivation rules of PVETO logic are extensions of axiomatic systems of propositional and epistemic logic and this in two different ways. First, on axioms and derivation rules that come from propositional and epistemic logic the derivation predicates P1,...,Pn of every player in the group are applied. Second, the temporal parameter is introduced as is explained in chapter 2.2. Asides from this, one new derivation rule and one new axiom are appended. So, axioms and derivation rules of PVETO logic are[2]:

Extensions of axioms from propositional logic[3]:
Pm(i(i(x,y),i(i(y,z),i(x,z))),w)
Pm(i(x,i(n(x),y)),w)
Pm(i(i(n(x),x),x),w)

Extension of Modus Ponens: -Pm(i(x,y),w) | -Pm(x,w) | Pm(y,w)

Extension of Distribution Axiom: Pm(i(Kp(i(x,y),v),i(Kp(x,v),Kp(y,v))),w)
Extension of Knowledge Axiom: Pm(i(Kp(x,v),x),w)

[1] In [5] and [9], symbol P ("provable") is a unary predicate symbol of the language of OTTER
[2] In every following axioms and derivation rules are Pm ∈ {P1,…,Pn} and Kp ∈ {K1,…,Kn}
[3] In this article we use axiomatic system of propositional logic according to Lukasiewicz [9, pp. 17]

Extension of Positive Introspection Axiom: Pm(i(Kp(x,v),Kp(Kp(x,v),v)),w)

Extension of Negative Introspection Axiom: Pm(i(n(Kp(x,v)),Kp(n(Kp(x,v)),v)),w)

Extension of Knowledge Generalization Rule: -Pm(x,w) | Pm(Km(x,w),w)

If the formula is derivable for player in moment w then it is derivable also in moment w+1 (derivation rule): -Pm(x,w) | Pm(x,$SUM(w,1))

Players never forget (axioms): Pm(i(Kp(x,v),Kp(x,$SUM(v,1))),w)

2.4. SPECIAL AXIOMS FOR THE GAME CLUE

In the game Clue all axioms and derivation rules of PVETO logic are valid. But the game Clue also has his own axioms. For a version with three active players and eight cards the axioms are as follow:

One card has exactly one player: Pm(i(n(d(ai,y)),i(n(d(aj,y)),i(n(d(ak,y)),d(al,y)))),w)
(ai, aj, ak, al ∈ {a0,a1,a2,a3}, i≠j≠k≠l)

Each player has one card which represents the suspect: Pm(i(d(x,c1),n(d(x,c2))),w)
(c1 and c2 are two different suspect cards)

Each player has one card which represents the weapon: Pm(i(d(x,c1),n(d(x,c2))),w)
(c1 and c2 are two different weapon cards)

2.5. THE END OF THE GAME

The game ends when at least one of the players knows the murderer and the murdering weapon. More formally, the end of the game is presented with statements that must be fulfilled in order for the game to finish. So, the game ends in the moment of time when at least one of the following pair of statements is true:

Pm(Km(d(a0,Professor_Plum),w),w)
Pm(Km(d(a0,Knife),w),w)

for m=1 or m=2 or m=3.

3. THE COURSE OF PLAY

In the moment t=0 players do not see their own cards, but they have knowledge in accordance with axioms of the game Clue. In the moment t=1 each player knows his own cards. According to basic propositions assumed in chapter 2.1, we can note following clauses:

P1(n(d(a1,Professor_Plum)),1)

P1(d(a1,Mr_Green),1)

P1(n(d(a1,Mrs_Peacock)),1)

P1(n(d(a1,Miss_Scarlett)),1)

P1(n(d(a1,Knife)),1)

P1(d(a1,Candlestick),1)

P1(n(d(a1,Pistol)),1)

P1(n(d(a1,Rope)),1)

P2(n(d(a2,Professor_Plum)),1)

P2(n(d(a2,Mr_Green)),1)

P2(d(a2,Mrs_Peacock),1)

P2(n(d(a2,Miss_Scarlett)),1)

P2(n(d(a2,Knife)),1)

P2(n(d(a2,Candlestick)),1)

P2(d(a2,Pistol),1)

P2(n(d(a2,Rope)),1)

P3(n(d(a3,Professor_Plum)),1)

P3(n(d(a3,Mr_Green)),1)

P3(n(d(a3,Mrs_Peacock)),1)

P3(d(a3,Miss_Scarlett),1)

P3(n(d(a3,Knife)),1)

P3(n(d(a3,Candlestick)),1)

P3(n(d(a3,Pistol)),1)

P3(d(a3,Rope),1)

In the moment t=2 player a1 makes a suggestion about murderer and weapon. He has many options as well as there are many options for actions during the course of play. Let us assume that the player a1 suggests cards Pistol and Professor_Plum. Afterwards players a2 and a3 know that player a1 has neither of these cards:

P2(K2(n(d(a1,Pistol)),2),2)

P2(K2(n(d(a1,Professor_Plum)),2),2)

P3(K3(n(d(a1,Pistol)),2),2)

P3(K3(n(d(a1,Professor_Plum)),2),2)

Player a3 also has neither of the suggested cards, he announces this and other players know that:

P1(K1(n(d(a3,Pistol)),3),3)

P1(K1(n(d(a3,Professor_Plum)),3),3)

P2(K2(n(d(a3,Pistol)),3),3)

P2(K2(n(d(a3,Professor_Plum)),3),3)

Now player a2 privately shows Pistol to the player a1. The clauses which represent the knowledge of players after this action are:

P1(K1(d(a2,Pistol),4),4)

P3(K3(i(n(d(a2,Pistol)),d(a2,Professor_Plum)),4),4)

P3(K3(i(n(d(a2,Professor_Plum)),d(a2,Pistol)),4),4)

We can assume the course of play after the moment t=4 (all knowledge of players after these actions can be represented by clauses): player a2 suggests Mr_Green and Candlestick, player a1 privately shows Mr_Green to the player a2, player a3 answers that he has neither card, player a3 suggests Mr_Green and Pistol, player a2 privately shows Pistol to the player a3, player a1 privately shows Mr_Green to the player a3, player a1 suggests Professor_Plum and Rope, player a3 privately show Rope to the player a1 and finally in the moment t=13 player a2 answers that he have neither card. It is provable that in the moment t=13 player a1 is a winner and players a2 and a3 know only the fact that murderer is Professor_Plum. We will prove these statements in the next chapter using OTTER.

4. REASONING USING OTTER

We build OTTER's input file in this way:

1. Axioms and derivation rules of PVETO logic as well as special axioms of the game Clue we put on the usable list[4]
2. Basic propositions and all statements about the knowledge of the players in singular moments of time we put on the sos list
3. Negations of statements that must be fulfilled in order for the game to finish in moment t=13 we put on the passive list
4. We use demodulation [5, pp. 307-359] to control temporal parameter in clauses. Inferred clauses must be discarded if they contain temporal parameter greater than 13. For example, demodulators for player a1 are:

P1(x,14) = junk

K1(x,14) = junk

n(junk) = junk

i(x,junk) = junk

i(junk,x) = junk

K1(junk,x) = junk

P1(junk,x) = $T

1. We use weighting [7, pp. 40-42] to assign low pick_given weights (high priorities) to long but important special axioms of the game Clue. For example, weight templates for player a1 are:

weight(P1(i(n(d($(1),$(1))),i(n(d($(1),$(1))),d($(1),$(1)))),$(1)),-100)

weight(P1(i(n(d($(1),$(1))),d($(1),$(1))),$(1)),-200)

[4] We use Sos strategy [5, pp. 78-81]

OTTER find all four proofs in 17 seconds. Here we show proof that player a1 at the moment t=13 knows that crime is committed with Knife:

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---------------- PROOF ----------------

10 [] -P1(i(x,y),w)| -P1(x,w)|P1(y,w).
22 [] P1(i(K1(x,v),x),w).
49 [] -P1(x,w)|P1(K1(x,w),w).
52 [] -P1(x,w)|P1(x,$SUM(w,1)).
81 [] P1(i(n(d(a2,y)),i(n(d(a3,y)),i(n(d(a1,y)),d(a0,y)))),w).
175 [] P1(i(d(x,Candlestick),n(d(x,Knife))),w).
178 [] P1(i(d(x,Pistol),n(d(x,Knife))),w).
181 [] P1(i(d(x,Rope),n(d(x,Knife))),w).
213 [] P1(d(a1,Candlestick),1).
240 [] P1(K1(d(a2,Pistol),4),4).
268 [] P1(K1(d(a3,Rope),12),12).
276 [] -P1(K1(d(a0,Knife),13),13).
299 [hyper,213,52,demod] P1(d(a1,Candlestick),2).
320 [hyper,299,52,demod] P1(d(a1,Candlestick),3).
356 [hyper,320,52,demod] P1(d(a1,Candlestick),4).
392 [hyper,356,52,demod] P1(d(a1,Candlestick),5).
428 [hyper,392,52,demod] P1(d(a1,Candlestick),6).
464 [hyper,428,52,demod] P1(d(a1,Candlestick),7).
500 [hyper,464,52,demod] P1(d(a1,Candlestick),8).
536 [hyper,500,52,demod] P1(d(a1,Candlestick),9).
572 [hyper,536,52,demod] P1(d(a1,Candlestick),10).
608 [hyper,572,52,demod] P1(d(a1,Candlestick),11).
644 [hyper,608,52,demod] P1(d(a1,Candlestick),12).
680 [hyper,644,52,demod] P1(d(a1,Candlestick),13).
719 [hyper,680,10,175] P1(n(d(a1,Knife)),13).
6828 [hyper,240,10,22] P1(d(a2,Pistol),4).
6833 [hyper,6828,10,178] P1(n(d(a2,Knife)),4).
7375 [hyper,6833,10,81] P1(i(n(d(a3,Knife)),i(n(d(a1,Knife)),d(a0,Knife))),4).
7460 [hyper,7375,52,demod] P1(i(n(d(a3,Knife)),i(n(d(a1,Knife)),d(a0,Knife))),5).
7484 [hyper,7460,52,demod] P1(i(n(d(a3,Knife)),i(n(d(a1,Knife)),d(a0,Knife))),6).
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7508 [hyper,7484,52,demod] P1(i(n(d(a3,Knife)),i(n(d(a1,Knife)),d(a0,Knife))),7).
7532 [hyper,7508,52,demod] P1(i(n(d(a3,Knife)),i(n(d(a1,Knife)),d(a0,Knife))),8).
7556 [hyper,7532,52,demod] P1(i(n(d(a3,Knife)),i(n(d(a1,Knife)),d(a0,Knife))),9).
7580 [hyper,7556,52,demod] P1(i(n(d(a3,Knife)),i(n(d(a1,Knife)),d(a0,Knife))),10).
7604 [hyper,7580,52,demod] P1(i(n(d(a3,Knife)),i(n(d(a1,Knife)),d(a0,Knife))),11).
7628 [hyper,7604,52,demod] P1(i(n(d(a3,Knife)),i(n(d(a1,Knife)),d(a0,Knife))),12).
9319 [hyper,268,10,22] P1(d(a3,Rope),12).
9324 [hyper,9319,10,181] P1(n(d(a3,Knife)),12).
9417 [hyper,9324,10,7628] P1(i(n(d(a1,Knife)),d(a0,Knife)),12).
9432 [hyper,9417,52,demod] P1(i(n(d(a1,Knife)),d(a0,Knife)),13).
9450 [hyper,9432,10,719] P1(d(a0,Knife),13).
9504 [hyper,9450,49] P1(K1(d(a0,Knife),13),13).
9505 [binary,9504.1,276.1] $F.
------------ end of proof -------------
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5. CONCLUSION

In this article we have shown that the card version of the game Clue can be modelled by PVETO logic - propositional multi-modal epistemic logic with temporal parameter adapted for reasoning with OTTER. The model of game is comprised of the axioms and derivation rules of PVETO logic as well as of the some special axioms of the game Clue. In order to be able to reason about some situations in the game we have assumed one course of play. By OTTER we have proven statements that must be fulfilled in order for the game to finish. In order to avoid combinatorial explosions we have used OTTER's Sos strategy, demodulation and weighting.

Some interesting questions for further research are: whether this approach can give answer on question what the optimal strategies for players in the game Clue are and whether this approach can be extended to incorporate common and distributed knowledge among the players. [4]

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